LEVEL 5 (Math Analysis) BLITZ 2014

1. The domain of the function $f(x)=\sqrt{\dfrac{x^2-3x+2}{x^2-4x+3}}$ is:

a. $(-\infty,1]\cup[1,\infty)$ b. $(-\infty,1)\cup(3,\infty)$ c. $(1,2)\cup(2,3)$

d. $(-\infty,1)\cup(1,2)\cup(3,\infty)$ d. none of these

2. i^{23} is

a. i b. $-i$ c. -1 d. 1 e. none of these

3. The period of the function $y=9\sin(4\pi x-5)+7$ is :

a. 2π b. $\dfrac{1}{2}$ c. $\dfrac{\pi}{2}$ d. 2 e. none of these

4. The exact value of $\cos(120^\circ)$ is :

a. $-\dfrac{1}{2}$ b. $-\dfrac{\sqrt{3}}{2}$ c. $-\dfrac{\sqrt{2}}{2}$ d. $-\sqrt{3}$ e. none of these

5. The exact value of $\tan\left(\dfrac{87\pi}{6}\right)$ is :

a. -1 b. $\sqrt{3}$ c. $\dfrac{\sqrt{3}}{3}$ d. $-\dfrac{\sqrt{3}}{3}$ e. none of these

6. If $x^4=81$, then the number of solutions (real or complex) to the equation are:

a. 4 b. 3 c. 2 d. 1 e. none of these

7. If $f(x)=\dfrac{3x+8}{2x-A}$ and $f(0)=2$, then A is:

a. -1 b. -3 c. 4 d. 2 e. none of these

8. If the positive angle θ is in standard position and the terminal side of the angle passes through the point $(4,-3)$, then the value of $\csc\theta$ is:

a. $-\dfrac{5}{3}$ b. $\dfrac{3}{5}$ c. $-\dfrac{5}{4}$ d. $-\dfrac{4}{5}$ e. none of these

9. An equivalent expression for cos(3x) is :

a. $3\sin x\cos x$ b. $4\cos^3 x - 3\cos x$ c. $-\cos x$

d. $\cos^3 x - \sin^3 x$ e. none of these

10. The range of the function $f(x) = -x^2 + 6x - 1$ is:

a. $(-\infty, 3]$ b. $(-\infty, 6]$ c. $(-\infty, 8]$ d. $(-\infty, 10]$

e. none of these

11. If $(a, -5)$ is a point on the graph of $y = x^2 + 6x$, then the possible values for a are:

a. 2 or 3 b. -2 or -3 c. 1 or 5 d. -1 or 5

e. none of these

12. The maximum value of the function $f(x) = 12\sin x - 5\cos x$ is:

a. 7 b. 12 c. 13 d. 17 e. none of these

13. For x in the interval $[0, 2\pi)$, the number of solutions to the equation $\cos^2 x - 2\cos x = 0$ is:

a. 2 b. 1 c. 4 d. 6 e. none of these

14. The lengths of three sides of a triangle are 15 feet, 10 feet, and 7 feet. The area (in square feet) of the triangle is:

a. $12\sqrt{6}$ b. $40\sqrt{77}$ c. 75 d. $5\sqrt{42}$ e. none of these

15. If $f(x) = \frac{1}{x^2}$, then $\frac{f(x+h) - f(x)}{h}$ where $h \neq 0$ is:

a. $\frac{-2x - h}{x^2(x+h)^2}$ b. $\frac{-2x}{x^2h(x+h)^2}$ c. $\frac{1}{h^2}$ d. $\frac{-2xh}{(x+h)^2}$

e. none of these

16. In a certain triangle, two of the sides measure 5 inches and 3 inches, and the angle formed by these two sides measures 120°. The length of the side opposite the 120° angle (in inches) is:

a. $\sqrt{19}$ b. 7 c. $\sqrt{34}$ d. $\sqrt{41.5}$ e. none of these

17. If $f(x)=\sqrt{x}+1$, then $(fof)(x)$ is:

a. $2\sqrt{x}+1$ b. $2\sqrt{x}+2$ c. $\sqrt{\sqrt{x}+\sqrt{x+1}}$ d. $\sqrt{\sqrt{x}+1}+1$

e. none of these

18. In a circle of radius 9 miles, a sector is subtended by an angle of 20°. The length (in miles) of the arc would be :

a. 180 b. $\dfrac{32{,}400}{\pi}$ c. π d. 1 e. none of these

19. The curve given by the parametric equations $x=\sec t$, $y=2\cos t$, $0\le t<\dfrac{\pi}{2}$ is a portion of:

a. a hyperbola b. a parabola c. an ellipse d. a circle

e. none of these

20. The oblique asymptote of the function $f(x)=\dfrac{x^2-x-12}{x+1}$ is :

a. $y=x-1$ b. $y=x-2$ c. $y=x-3$ d. $y=x-4$

e. none of these

21. Let P be a point on the graph $y=x^2$. The expression that represents the distance from P to the point (0, 0) is:

a. $\sqrt{x^2+1}$ b. $|x|\sqrt{x^2+1}$ c. x^2+x d. $x^2\sqrt{x^2+1}$

e. none of these

22. When $2x^{25}-3x^{12}+5x-1$ is divided by $x+1$, the remainder is :

a. 1 b. -1 c. 11 d. -11 e. none of these

23. If $A=\begin{bmatrix}1 & 3\\ -2 & 1\end{bmatrix}$, then A^2 is:

a. $\begin{bmatrix}2 & 6\\ -4 & 2\end{bmatrix}$ b. $\begin{bmatrix}1 & 9\\ 4 & 1\end{bmatrix}$ c. $\begin{bmatrix}-5 & 6\\ -4 & -5\end{bmatrix}$ d. undefined

e. none of these

24. The exact value of $\cos(157.5^\circ)$ is :

a. $\frac{\sqrt{2}}{4}$ b. $\frac{1}{2}\sqrt{2+\sqrt{2}}$ c. $-\frac{1}{2}\sqrt{2-\sqrt{2}}$ d. $-\frac{1}{2}\sqrt{2+\sqrt{2}}$

e. none of these

25. If $f(x)=\frac{x+1}{x-3}$, then $f^{-1}(x)$ is:

a. $\frac{x-3}{x+1}$ b. $\frac{3x+1}{x-1}$ c. $\frac{x-1}{x+3}$ d. $\frac{x+1}{3x-1}$

e. none of these

26. The function $f(x)=-e^{|x|}$ is increasing on the interval :

a. $(-\infty,0]$ b. $(-\infty,1]$ c. $[0,\infty)$ d. $[1,\infty)$

e. none of these

27. The trigonometric form of the vector $-i+j$ is:

a. $\sqrt{2}\left((\cos 45^\circ)i+(\sin 45^\circ)j\right)$ b. $-\sqrt{2}\left((\cos 45^\circ)i+(\sin 45^\circ)j\right)$

c. $\sqrt{2}\left((\cos 135^\circ)i+(\sin 135^\circ)j\right)$ d. $2\left((\cos 315^\circ)i+(\sin 315^\circ)j\right)$

e. none of these

28. The value of $\cos\left[2\cos^{-1}\left(\frac{\sqrt{3}}{3}\right)\right]$ is:

a. $-\frac{1}{3}$ b. $-\frac{2\sqrt{3}}{3}$ c. $\frac{\sqrt{6}}{3}$ d. $\frac{2\sqrt{3}}{3}$ e. none of these

29. Let $\bar{u}=\bar{i}-2\bar{j}$ and $\bar{v}=-3\bar{i}+2\bar{j}$. If $\bar{R}=6\bar{u}-2\bar{v}$, then the magnitude of $\bar{R}$ is:

a. $12\sqrt{2}$ b. $8\sqrt{5}$ c. $6\sqrt{10}$ d. 20 e. none of these

30. The exact value of $2^{\log_2 3+\log_2 5}$ is :

a. 4 b. 8 c. 15 d. 256 e. none of these

31. The expression $(\log_2 5)(\log_5 10)(\log_{10} 16)$ is equal to :

a. $1+\log_{100} 8$ b. $\log_{10} 31$ c. $\log_{100} 31$ d. 4

e. none of these

32. The sum of the solutions to the equation $3^{x^2} = \frac{9}{3^x}$ is :

a. -2 b . -1 c. 0 d. 1 e. none of these

33. The focus of the parabola $y^2 + 4y = -8x - 12$ is:

a. $(-3,2)$ b. $(-3,-2)$ c. $(-1,2)$ d. $(-1,-2)$

e. none of these

34. A hyperbola has foci at $(\pm 2,0)$ and asymptotes $y = \pm x$. The distance between the vertices of this hyperbola is:

a. $2\sqrt{2}$ b. 2 c. $4\sqrt{2}$ d. 4 e. none of these

35. For x in the interval $[0,2\pi)$, the number of solutions to the equation $\sec^2(3x) = 2$ is:

a. 0 b. 4 c. 8 d. 12 e. none of these

36. The sum of the infinitive series $\sum_{k=1}^{\infty} 2\left(\frac{3}{4}\right)^k$ is:

a. $\frac{3}{4}$ b. 6 c. 12 d. 24 e. none of these

37. The y-intercept of the line that is perpendicular to the line with equation $2x + y = 2$ and passing through the point $(-1,4)$ is:

a. $(0,2)$ b. $(0,-2)$ c. $\left(0,\frac{9}{2}\right)$ d. $\left(0,-\frac{9}{2}\right)$

e. none of these

38. If $\vartheta = Cos^{-1}\left(\frac{x}{3}\right)$, then the expression $\frac{\sqrt{9-x^2}}{x}$ can be simplified as:

a. $\tan(\vartheta)$ b. $3\cot(\vartheta)$ c. $\frac{1}{3}\tan(\vartheta)$ d. $3\sin(\vartheta)$

e. none of these

39. The graph of the polar equation $r = 3\cos(4\theta)$ for $0 \le \theta \le 2\pi$ is :

a. a cardiod b. a circle c. a four-leafed rose

d. an eight-leafed rose e. none of these

40. If $\sec(\beta) = -1.2$ and $\frac{\pi}{2} < \beta \le \pi$, then $\csc(\beta)$ equals :

a. $-\frac{5}{6}$ b. $\frac{\sqrt{11}}{6}$ c. $-\frac{6}{\sqrt{11}}$ d. $\frac{6}{\sqrt{61}}$ e. none of these

41. The value of the sum $19 + 25 + 31 + ... + 1{,}213$ is :

a. 12,310 b. 31,210 c. 123,200 d. 212,300

e. none of these

42. A ball is dropped from a height of 10 feet. Each time it hits the ground, it rebounds to 75% of its previous height. The total distance traveled by the ball is :

a. 50 feet b. 60 feet c. 70 feet d. 80 feet

e. none of these

43. If α is an acute angle and $\sin(\alpha) = \frac{3}{5}$, then the sine of the supplement of α is:

a. $\frac{3}{5}$ b. $-\frac{3}{5}$ c. $\frac{4}{5}$ d. $-\frac{4}{5}$ e. none of these

44. Counting the number of the ordered pair solutions to the system $\begin{cases} x^2 + y^2 = 4 \\ y = 3 - x^2 \end{cases}$ is:

a. 0 b. 1 c. 2 d. 3 e. none of these

45. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$, then A^{-1} is :

a. $\begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$

c. $\begin{bmatrix} -2 & 1 \\ \frac{5}{2} & -\frac{3}{2} \end{bmatrix}$

d. $\begin{bmatrix} \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}$

e. none of these